FRIEDRICH-SCHILLER-UNIVERSITÄT JENA

Department of Mathematics
Stochastic Analysis (SS 2019)

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Exercise sheet 2

Submission: 23.04.2019

Problem 1 **(5 Points)**

(a) Let $(X_n)_{n\geq 0}$ be an $(\mathcal{F}_n)$-martingale which converges to a random variable X in L^p for some $p \geq 1$. Show that

$$X_n = \mathbb{E}[X \,|\, \mathcal{F}_n] \quad \text{a.s.}$$

(b) Let $(Z_k)_{k\geq 1}$ be a sequence of i.i.d. random variables with $Z_1 \sim \mathcal{N}(0,1)$. Define $S_0 = 0$ and $S_n = \sum_{k=1}^{n} Z_k$, $n \geq 1$. For an $\alpha \in (0,\infty)$ consider $X_n := \exp(\alpha S_n - \frac{\alpha^2}{2} n)$ and $\mathcal{F}_n := \sigma(S_0, S_1, ..., S_n)$, $n \geq 0$. Show that $(X_n)_{n\geq 0}$ is a martingale w.r.t. $(\mathcal{F}_n)_{n\geq 0}$. Does $(X_n)_{n\geq 0}$ converge a.s. and in L^1?

Total: 5 Points

Terms of submission:

- Solutions can be submitted in groups of at most 2 students.
- Please submit at the beginning of the lecture or until 9:50 a.m. in room 3523, Ernst-Abbe-Platz 2.